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Approximation of a function describing a quality criterion of the thermo-mechanical fusible interfacing process

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ABSTRACT

The present work aims to investigate the function describing the relationship between a quality criterion and input factors of the thermo-mechanical fusible /TMF/ interfacing process and to derive its effective approximation. An approximation by interpolation was applied for the purpose of the study.

A numerical realization of a linear and exponential approximation of the mathematical model describing the TMF interfacing process was performed. An effective linear approximation of the function connecting the quality criterion with the input factors of the TMF interfacing process was found. This creates conditions for replacing the relatively complex function (describing the TMF interfacing process) with its linear approximation. The linear approximation gives the possibility easier and faster to determine the relationships between the input factors and the quality criterion. This created conditions for ignoring the subjective factor and for optimizing and automating the studied technological process.

Keywords
approximation,
thermo-mechanical
fusible interfacing process,
quality criterion

1 Introduction

Nowadays, the achievements of mathematics allow the application of mathematical modeling for various objects and processes. Mathematical modeling and optimization acquires special significance in the modern conditions of accelerated scientific and technical progress, in the need to achieve high efficiency with limited financial, material, labor, energy and time resources.

The mathematical methods for analysis, modeling and optimization are increasingly used in the sewing and textile technologies [1-4]. This allows avoiding the subjective factor and creates real conditions for automation of the processes.

One of the effective methods for study a given function is the method of approximation [5-7].

With this method the investigation of various (unknown or extremely complex) numerical characteristics and qualitative properties of the original objects reduces to working with other objects whose characteristics and properties are already known or more convenient to work with [5-7].

After the analysis of the literature survey, it can be summarized that the methods of approximation are applied to a number of technological processes in the textile and clothing industry [8-12].

For example, [8] presents the mathematical and computer simulation of multiparameter systems. The simulation is based on experimental data and is achieved by modifying the one-dimensional approximations of splines. The method is used in the study of some technological conditions of fabric lamination systems and gives good results.

In [9] mathematical and computer models are developed for predicting effective elastic properties for a complex periodic cell and a representative volume (fragment) of spatially amplified composite material (SRCM). Numerical experiments are performed to predict the effective elastic properties of a cell with SRCM periodicity with an orthogonal tissue pattern with two variants of the structure and the representative volume cell using the local approximation method.

An approximation of a mathematical model of the thermo-mechanical fusing process is proposed in [13].

The thermo-mechanical fusing process is one of the main technological processes in the sewing production. The quality and productivity achieved in this process significantly affect the quality and productivity of the entire production in the sewing company. Therefore, it is important to investigate the thermo-mechanical fusing process, to create mathematical models describing the process [14,15], and to search for opportunities for their approximation [13].

In [13], an approximation of the function connecting an output criterion for performance with input factors of the thermo-mechanical fusing process is proposed.

It is especially important to study and analyze an output quality criterion as well. Several studies [16,17, 18] analyze the influence of input factors on various quality criteria of the thermo-mechanical fusing process. In [15] a function is derived connecting an output quality criterion with factors influencing the fusing conditions.

Finding an approximation of this function [15] which describes the thermo-mechanical fusible /TMF/ interfacing process in a simpler way is of particular interest.

This work aims to investigate the function describing the relationship between a quality criterion and input factors of the TMF interfacing process and to derive its effective approximation.

2 Research work

2.1 Methods

The standard algorithm for approximating functions, described in detail in [5, 6, 7, 13], is used in the research.

Taking into account the conditions for conducting the present studies, interpolation is used as a method of approximation [5, 6, 7].

The main steps in applying the interpolation method are [5, 6, 7]:

- let the function $y = f(x)$ be defined in some interval and a table of its values be known for the respective x_i [x, ш];

- an approximate function $\varphi(x)$ is sought such that

$\varphi(x_i) = y_i, i = 0,1, \dots, n.$ (1)

This problem is solved by setting the class of functions $\varphi(x)$.

Graphically, condition (1) means that the approximating function $\varphi(x)$ passes through the points (x_i, y_i), since x_i has the same values as $f(x)$ (Figure 1).

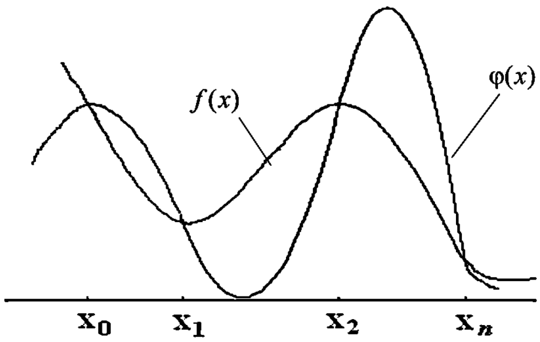

Fig. 1 Indicative graphical representation of the condition (1).

After performing the interpolation, it is necessary to determine the coefficient of determination R^2 [6]. It measures how much of the approximation error is eliminated. The coefficient of determination is a measure of the quality of the approximation model and varies in the range [0;1], or in percentages [0; 100]. The closer R^2 is to 1 (up to 100%), the greater the efficiency of the approximation model.

Specialized software Maple and Matвab is used for the research in the present work.

2.2 Materials

Materials produced by the company NITEX-50 - Sofia were used for basic textile materials /TM/. They are 100% wool fabrics. Their characteristics are described in detail in [14].

Material produced by the company Kufner-B121N77 was used for an auxiliary TM (interlining). Their characteristics are described in detail in [14] as well.

2.3 Conditions for conducting the study

For the numerical realization of the research the experimentally obtained function (2) is used [15]:

$$Y = 1.255 + 0.3225.x_1 + 0.21.x_2 + 0.155.x_3 - 0.0175.x_1.x_2 - 0.04.x_2.x_3 + 0.0175x_1.x_2.x_3 \quad (2)$$

The function (2) describes the relationship between a quality criterion Y (The change of the color shade of the TM after TMF interfacing process) and manageable process factors X_1 – Pressure, P [N/cm^2], X_2 - Temperature of the pressing plates, T [°C] , X_3 - Mass per unit area of the basic textile materials, M [g/m^2].

The correlation field from the experimental data that are processed with the specialized software Maple and MatLab is used.

The best approximation to the experimentally derived function (2) is sought. An approximation of function (2) is applied by interpolation in linear and exponential form. The investigations were performed with the coded values of the factors. The relationship between the natural and coded values of the factors is given in [14]. Three variants are investigated. In each variant, one of the factors assumes values in the range [-1; +1], and the other two factors are constants. The constant values are the values of the factors at which the function $Y(X_1, X_2, X_3)$ is optimal.

The optimal value of the selected quality criterion $Y_{opt} = Y_{min} = 0.4975$ is reached at the following values of the input factors: the pressure P = 10 [N/cm^2], the temperature of the pressing plates T = 120 [°C] and the mass per unit area of basic textile materials M = 173 [g/m^2] [18]. The coded values of the factors in which Y_{min} is obtained are $X_1 = (-1)$; $X_2 = (-1)$; $X_3 = (-1)$ [18].

A linear and exponential approximation of the function (2) is made for the following three variants:

- variant I - $X_1 \in [-1;1];\ X_2 = (-1);\ X_3 = (-1);$

- variant II - $X_2 \in [-1;1];\ X_1 = (-1);\ X_3 = (-1);$

- variant III - $X_3 \in [-1;1];\ x_1 = (-1);\ x_2 = (-1).$

3 Results and discussion

3.1 Results of the approximation

The numerical results of the linear and exponential approximation of the function (2) for the first variant are given in Table 1.

Table 1. Numerical results of the approximations of the function (2) for variant 1.

$\mathbf{X_1}$	**Linear Approximation** $\mathbf{Y = 0.93 + 0.3225x_1}$	**Exponential Approximation** $\mathbf{Y = 0.9059e^{0.3577x_1}}$
-1.00	0.607500	0.633481
-0.75	0.688125	0.692739
-0.50	0.768750	0.757542
-0.25	0.849375	0.828406
0.00	0.930000	0.905900
+0.25	1.010625	0.990643
+0.50	1.091250	1.083313
+0.75	1.171875	1.184652
+1.00	1.252500	1.295470

The graphical results of the linear and exponential approximation of the function (2) for the first variant are presented in figure 2.

For 1st variant: the linear and exponential approximation coincide, i.e.

$$Y_{LinAppr} = Y_{ExpAppr}, \tag{3}$$

where:

$$Y_{LinAppr} = Y = 0.93 + 0.3225x_1 \tag{4}$$

$$Y_{ExpAppr} = Y = 0.9059e^{0.3577x_1}, \tag{5}$$

when:

$$0.93 + 0.3225x_1 = 0.9059e^{0.3577x_1} \tag{6}$$

only for values for X_1:

$X_1 = -0.6857666234;$

$X_1 = 0.6092238139.$

The function Y (for these values for X_1) takes values accordingly:

$$Y(X_1 = -0.6857666234) = 0.7088402640 \tag{7}$$

$$Y(X_1 = 0.6092238139) = 1.126474680 \tag{8}$$

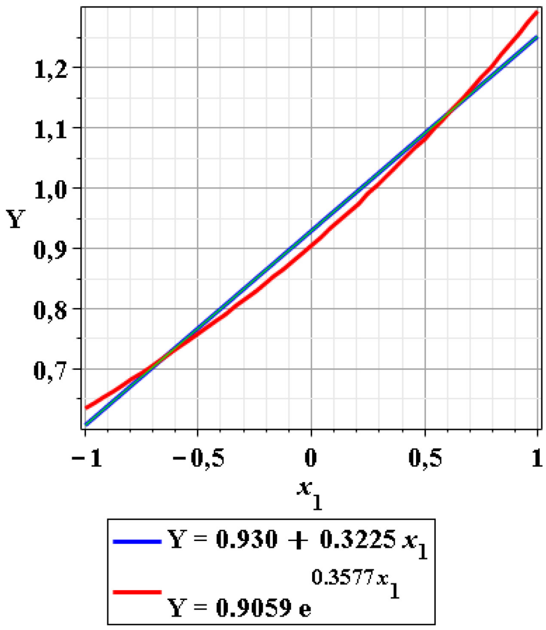

Fig. 2 Graphical results of the approximations of the function (2) for variant 1.

The numerical results of the linear and exponential approximation of the function (2) for the second variant are given in Table 2.

Table 2. Numerical results of the approximations of the function (2) for variant 2.

X_2	**Linear Approximation** $\mathbf{Y = 0.7775 + 0.2500x_2}$	**Exponential Approximation** $\mathbf{Y = 0.7603e^{0.3302x_2}}$
-1.00	0.52750	0.546488
-0.75	0.59000	0.593515
-0.50	0.65250	0.644589
-0.25	0.71500	0.700058
0.00	0.77750	0.760300
+0.25	0.84000	0.825726
+0.50	0.90250	0.896782
+0.75	0.96500	0.973953
+1.00	1.02750	1.057765

The graphical results of the linear and exponential approximation of the function (2) for the second variant are presented in figure 3.

For 2nd variant: the linear and exponential approximation coincide, i.e.

$$Y_{LinAppr} = Y_{ExpAppr}, \quad (9)$$

where:

$$Y_{LinAppr} = Y = 0.7775 + 0.2500x_2 \quad (10)$$

$$Y_{ExpAppr} = Y = 0.7603e^{0.3302x_2}, \quad (11)$$

when:

$$0.7775 + 0.2500x_2 = 0.7603e^{0.3302x_2} \quad (12)$$

only for values for X_2:

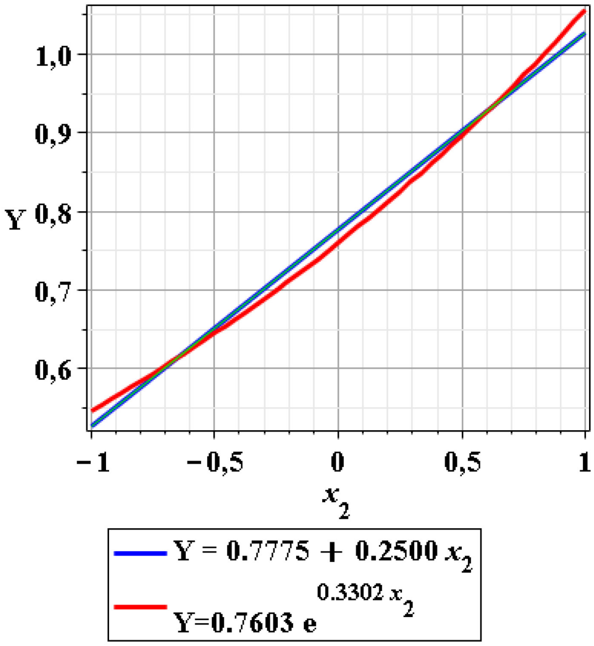

Fig. 3 Graphical results of the approximations of the function (2) for variant 2.

$X_2 = -0.6821481353;$

$X_2 = 0.6108059334.$

The function Y (for these values for X_2) takes values accordingly:

$$Y(X_2 = -0.6821481353) = 0.6069629662 \quad (13)$$

$$Y(X_2 = 0.6108059334) = 0.9302014834 \quad (14)$$

The numerical results of the linear and exponential approximation of the function (2) for the third variant are given in Table 3. The graphical results of the linear and exponential approximation of the function (2) for the third variant are presented in figure 4.

For 3rd variant: the linear and exponential approximation coincide, i.e.

$$Y_{LinAppr} = Y_{ExpAppr}, \quad (15)$$

Table 3. Numerical results of the approximations of the function (2) for variant 3.

X_3	**Linear Approximation** $Y = 0.74 + 0.1775x_3$	**Exponential Approximation** $Y = 0.731e^{0.2434x_3}$
-1.00	0.56250	0.573073
-0.75	0.60688	0.609028
-0.50	0.65125	0.647238
-0.25	0.69563	0.687845
0.00	0.74000	0.731000
+0.25	0.78438	0.776863
+0.50	0.82875	0.825603
+0.75	0.87313	0.877400
+1.00	0.91750	0.932448

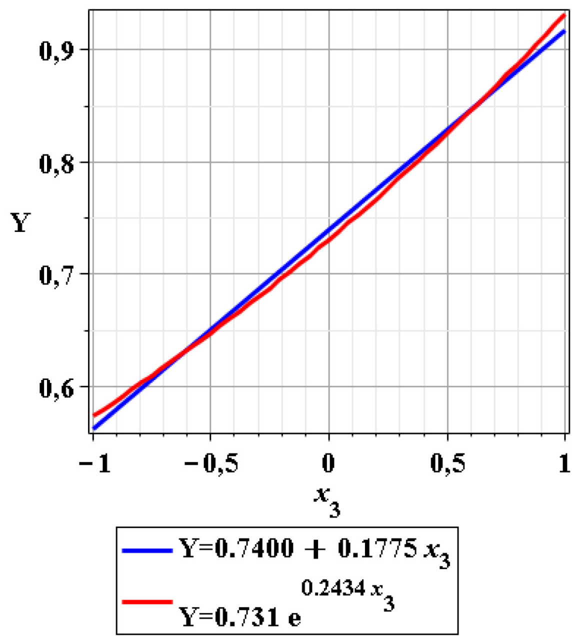

Fig. 4 Graphical results of the approximations of the function (2) for variant 3.

where:

$$Y_{LinAppr} = Y = 0.74 + 0.1775x_3 \quad (16)$$

$$Y_{ExpAppr} = Y = 0.731e^{0.2434x_3}, \quad (17)$$

when:

$$0.74 + 0.1775x_3 = 0.731e^{0.2434x_3} \quad (18)$$

only for values for X_3:

$X_3 = -0.6727381075;$

$X_3 = 0.6192396062.$

The function Y (for these values for X_3) takes values accordingly:

$$Y(X_3 = -0.6727381075) = 0.6205889859 \quad (19)$$

$$Y(X_3 = 0.6192396062) = 0.8499150301 \quad (20)$$

The summarized numerical results for the value of the investigated function (2) for linear and exponential approximation are presented graphically in Figure 5.

3.2 Discussion of the obtained numerical results

The investigation shows that the optimal value of the function Y is reached at a point with coordinates (-0.9999;-1;-1) and this value is 0.49249999, i.e.,

$$Y_{min} = Y(X_1 = -0.99999999999999988; X_2 = -1; X_3 = -1) = 0.49249999 \quad (21)$$

The differences in the values at the local minimum of the studied function (2) and the local minima of the function in the considered variants are insignificant of the order of at most 10^{-1}.

This is due to rounding one of the variables X_1 from -0.99999999999999988 to -1. (The used software rounds the data.)

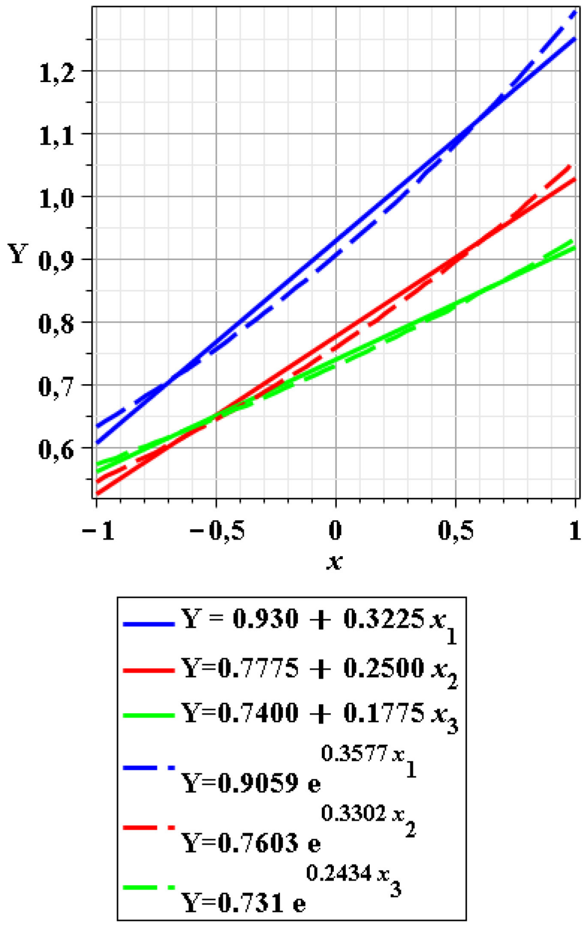

Fig. 5 Movement of the value of the objective function Y.

For each of the considered variants the coefficient of determination R^2 is determined /table 4/ for the linear and exponential approximation of the function (2) respectively. The results for R^2 show the high efficiency of both approximation models /linear and exponential/.

The data in Table 4 mean that:

- the model in the linear approximation explains between 99.07% and 99.63% of the experimental data;

- the exponential approximation model explains between 98.96% and 99.52% of the experimental data.

Therefore, the linear approximation of the investigated function (2) is more efficient.

Table 4. Values of the coefficient of determination R^2 in the linear and the exponential approximation.

	1st variant	2nd variant	3rd variant
$R^2_{LinAppr}$	0.9907	0.9932	0.9963
$R^2_{ExpAppr}$	0.9896	0.9912	0.9952

4 Conclusions

The present work investigates the nature of a function describing the TMF interfacing process. The function gives the relationship between the quality criterion Y (the change of the color shade of the textile material after TMF interfacing process) and the input factors X_1 - the pressure P, [N / cm^2]; X_2 - the temperature of the pressing plates T, [° C]; X_3 - the mass per unit area of basic textile materials M, [g / m^2] [15]. An approximation by interpolation was applied for the purpose of the study.

A numerical realization of a linear and exponential approximation of the mathematical model describing the TMF interfacing process was performed. Three generalized variants with different values of the input factors were taken into consideration. For each of the studied variants, the corresponding values for the change of the color shade of the TM after TMF interfacing process were obtained.

An effective linear approximation of the function connecting the quality criterion with the input factors of the TMF interfacing process was found. This creates conditions for replacing the relatively complex function (describing the TMF interfacing process) with its linear approximation. The linear approximation makes it easier and faster to determine the relationships between the input factors and the quality criterion. This is of essential importance for the quality and efficiency of the TMF interfacing process.

Of course, the proposed mathematical model of the process and its linear approximation can be applied to the described operating conditions / for the respective type of press used, for the respective types of textile materials, etc./. The principles and methods of research of TMF interfacing process / used in the present work / can also be applied in the research of other textile materials, when working with another type of press, etc.

It can be summarized that the proposed methodology for research and analysis of the TMF interfacing process is applicable to different operating conditions.

In the present work, a specific technological process was studied and analyzed using mathematical methods and modern software products. An effective approximation of a mathematical model of the process was applied. This created conditions for facilitating the work, for ignoring the subjective factor and for optimizing and automating the studied technological process.

References

[1] Gries, T.; Lutz, V.; Niebel, V.; Saggiomo, M.; Simonis, K. 14: Automation in quality monitoring of fabrics and garment seams. In *Automation in Garment Manufacturing*, 1st ed.; Rajkishore, N., Rajiv, P.; Woodhead: 2018, pp. 353-376. https://doi.org/10.1016/B978-0-08-101211-6.00014-8

[2] Watcharapanyawong, K.; Sirisoponsilp, S.; Sophatsathit, P. A Model of Mass Customization for Engineering Production System Development in Textile and Apparel Industries in Thailand. *Systems Engineering Procedia,* 2011, 2, pp. 382-397. https://doi.org/10.1016/j.sepro.2011.10.052

[3] Quezada-García, S.; Espinosa-Paredes, G.; Polo-Labarrios, M.; Espinosa-Martínez, E.; Escobedo-Izquierdo, M. Green roof heat and mass transfer mathematical models: A review. *Building and Environment*, 2020, 170, Article 106634. https://doi.org/10.1016/j.buildenv.2019.106634

[4] Dimitrienko, Yu. I.; Dimitrienko, I. D. Modeling and simulation of textile composite plates under a punching load. *Applied Mathematics and Computation,* 2020, 3641, Article 124660. https://doi.org/10.1016/j.amc.2019.124660

[5] Agarwal, B.L. *Basic statistics*, e-book, 4th ed.; New Age International (P) Limited: 2006; pp. 523–563. https://books.google.bg/books?id=qT2srNLLxB0C&printsec=frontcover#v=onepage&q&f=false

[6] Giri, P.; Banerjee, J. *Introduction to statistics*, e-book, 6th ed.; Academic Publishers: 2008; pp. 1-90. https://books.google.co.in/books?id=1CiL9H7mO7wC&printsec=frontcover#v=onepage&q&f=false

[7] http://db.cs.duke.edu/courses/fall01/cps150/lectures/Interpolation.pdf (27 July 2020).

[8] Chizhik, M.; Yurkov, V.; Litunov, S.; Skuba, P. Nonlinear net models of multi-parametrical systems and processes. Journal of Physics: Conference Series of XIII International Scientific and Technical Conference "Applied Mechanics and Systems Dynamics", Omsk, Russian Federation, IOP: 2019; 1441. https://iopscience.iop.org/article/10.1088/1742-6596/1441/1/012053/pdf

[9] Anoshkin, Al.; Pisarev, P.; Ermakov, D.; Roman, Ks. Forecasting Effective Elastic Properties of Spatially Reinforced Composite Materials Applying the Local Approximation Method. AIP Conf. Proc. 2216 of the 28th Russian Conference on Mathematical Modelling in Natural Sciences, Perm, Russia, 2–5 October 2019; Matveenko, Trusov, Yants, Faerman; AIP Publishing: 2020, 020008-1–020008-7, https://doi.org/10.1063/5.0004078

[10] Kosheleva, M.; Golykh, R.; Novikova, T.; Dorovskikh, R.; Khmelev, V.; Shalunov, A. Ultrasonic drying of textile materials. Proceedings of 18th International Conference of Young Specialists on Micro/Nanotechnologies and Electron Devices, Erlagol, Russia, 29 June-3 July 2017; IEEE: 2017; INSPEC Accession Number: 17042461. https://doi.org/10.1109/EDM.2017.7981757

[11] Yu-xian, Zhang; Yong, Li; Bin, Cheng. A nonlinear approximation for size moisture regain using moving least squares method. Proceedings of 24th Chinese Control and Decision Conference, Taiyuan, China, 23-25 May 2012; IEEE: 2012; INSPEC Accession Number: 12881023. https://doi.org/10.1109/CCDC.2012.6244142

[12] Kosheleva, M.; Dornyak, O.; Novikova, T.; Tsintsadze, M. Approximation of Sorption Isotherms of Fiber Materials by Various Equations. *Fibre Chemistry*, 2019, 51, 268–271. https://doi.org/10.1007/s10692-020-10092-y

[13] Andonova, Sn.; Baeva, S. Approximation of a mathematical model of the thermo-mechanical fusing process in the sewing industry. *Fibres and textiles* under review.

[14] Andonova, Sn. Mathematical modelling of the thermo-mechanical fusing process. *Fibres and textiles*, 2020, 2, 3-7.

[15]Andonova, Sn. An influence of fusing conditions on the change of color shade in the production of clothing. *Tekstilec* accepted.
[16]Motejl, V. *Machines and equipment in clothing production*; SNTL: Praha, Czech Republic, 1984; pp. 206-213.
[17]Andonova, S.; Stefanov, A.; Amudzhev, I. An investigation of the thermo-mechanical fusing process of innovative textile material. *Industria Textila Journal* accepted.
[18]Andonova, Sn.; Baeva, S. Optimizing a function linking an quality criterion to input factors on the thermo-mechanical fusing process. *Fibres and textiles* under review.