

Developing lasts with removable toe parts for customized footwear

Liliia Chertenko^{1,*}, Tatjana Spahiu², Tymofii Lypskiy¹, Henrique Almeida³, Olexander Bondar¹

¹Kyiv National University of Technologies and Design, Ukraine

²Polytechnic University of Tirana, Albania

³Polytechnic Institute of Leiria, Portugal

*Corresponding author E-mail address: lily-che@ukr.net

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ABSTRACT

The scope of this work is to develop lasts with a removable toe part as one of the ways to optimize the range of lasts while maintaining a high level of conformity of the last shape to the individual measurements of the feet of customers. For this purpose, the sectional plane separating the toe part of different shapes and styles from the main back part, which corresponds to the measurements of the foot, is determined by the area of minimal deviations of the parameters of feet shapes and lasts in different 3D models. The interchange of toe parts between different lasts is carried out under the condition of a small difference between their major parameters when it is possible to perform the averaging of the shapes and dimensions of the sectional plane of different lasts without reducing the ergonomics of their shapes. The segmentation plane divides the last body in such a way that the removable toe parts can be attached to the main back parts of the lasts of different styles and forms, as well as with different heel heights, which increases the use efficiency of the replaceable elements.

Keywords

3D modeling,
shoe last,
toe part,
last back part,
foot measurements,
morphing,
blending

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1 Introduction

The shoe comfort is primarily linked with the ergonomic shape of the last. With regard to this requirement, the worst situation is observed in the field of elegant and fashionable shoes. These shoes are not very comfortable and can worsen postural control [1], but not in the case of personalized heels [2]. Despite this, people wearing heels above the height of 2 inches are more prone to developing footwear problems such as forefoot pain [3]. To increase the comfort level of high-heeled shoes, it is

necessary to improve the shape of the crank profile and footbed shape [4], as well as to use additional shoe inserts [5].

A good compromise between a fashionable design and a comfortable shape can be the development of an ergonomic main part of a last (using scientifically grounded rationale parameters) which will serve as the basis for creating new shapes by changing the design of the toe part because it is the toe part of the shoe last that changes its configuration under the influence of new fashion trends.

The use of advanced technologies in footwear products such as 3D scanning confirms the existence of many statistically significant differences in mean foot measurements among the regions and between the genders [6]. Meanwhile, digital methods for taking anthropometric data are more accurate and the personalization process is faster compared to traditional methods [7-8]. These technologies help to design complex shapes of the lasts in the 3D virtual space. As a result, 3D technology help to overcome problems encountered in footwear personalization [9-11] by improving the manufacturing of footwear products [12-13]. 3D scanning of the consumer's foot can be used to update shoe sizing tables for the last design [14] by reducing the number of manual and repetitive operations [15,16]. Virtual last models can be used to create a personalized library of the shoe lasts [17]. 3D technology has become popular due to the convenience of virtual presentation, easy modifications, and evaluation of functional properties of footwear [18]. Moreover, the cases for orthopedic and sports shoes are of great interest due to the specifics that these products should have. Regarding elegant fashion shoes, here, for example, Italian trendsetters in the footwear industry emphasize the benefits of the traditional manual approach for shoe design and production, when the first last sample is handmade by an artist-last maker [16]. With this approach, the standard sample of the last made by the last maker has to be adopted by the manufacturers to the foot measurements and physical needs of a consumer.

Studies of the footwear fitting for the people of the studied categories show between 63% and 72% of participants were wearing shoes that did not accommodate either in width or length dimensions of their feet [19]. There are many different problems related to tightness (fitness) as the width of the shoe which is an important part of fit [20]. One of the main indicators of the quality of shoes is their comfort, which mostly depends on the proper design of the shoe last [21,22]. The improper shape of the last is a major factor in the development of structural disorders of the foot such as hallux valgus and the deformity of toes, as well as skin lesions such as calluses and ulcers. Meanwhile, designing lasts with foot dimensions and taking into account the foot shape enables improving the compliance and fitness of footwear [23].

In custom-made shoe manufacturing, the number of last shapes and sizes increases due to a large variety of customers' feet as well as different shoe styles. Transforming lasts with an ergonomic basic back part and a fashionable replaceable toe part can significantly increase the rationalization of production and become a method that allows putting into practice the concept of universal lasts that can be used for many years. In custom-made shoe manufacturing, such a solution is difficult to implement because of the huge variety of shapes and sizes. The objective of this work is to find conditions to ensure mutual exchange of removable toe parts between the lasts of similar dimensional parameters.

2 Methodology

A group of 12 females from 20 to 35 years with foot length (245.0 ± 2.5) mm and ball girth (229 ± 6) mm are selected. From a warehouse of the bespoke shoe production 12 women's shoe lasts of different styles for size 38 (EU) and standard width are selected. The heel heights of the selected shoe lasts are: 15 mm (2 models), 30 mm (3 models), 50 mm (3 models), 80 mm (2 models), or 90 mm (2 models).

The feet and lasts are scanned using the InFoot 3D (OrthoBaltic, Lithuania), a specialized 3D scanner with an accuracy of 0.3 mm (Figure 1). During the scanning process, the person stands on their feet and the bodyweight is distributed evenly over the two legs. Both the right and left feet are scanned. In the case of participants with high-heel shoes, they are scanned in two positions: standard (full-foot support)

and with high heel shoes. Data processing with automatic measurement of the main anthropometric parameters of the feet is performed in the Foot 3D special software. The basic graphical processes are realized on PowerShape CAD, which has a wide range of features for surface and solid-state modeling. The data taken are 3D feet and last models as polygonal mesh, footprints, and the main anthropometric parameters.

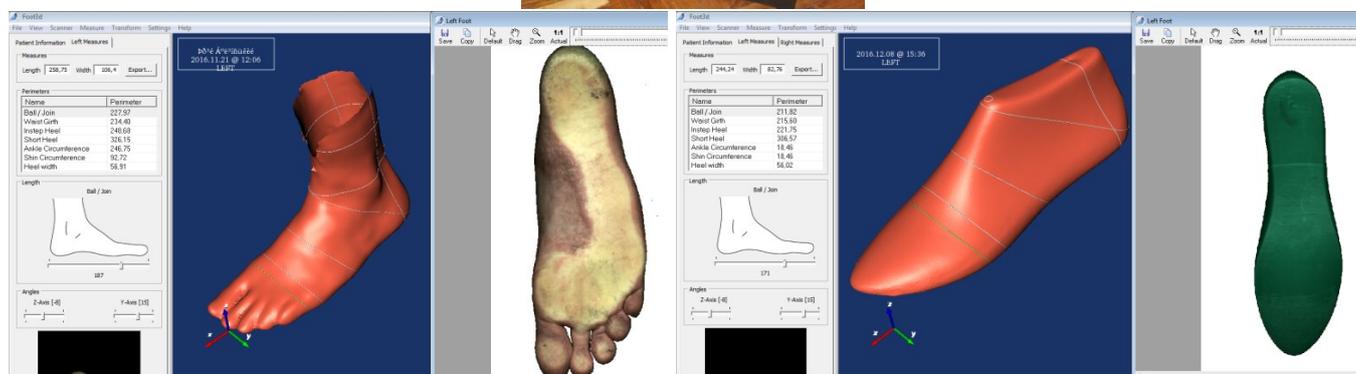


Fig. 1 Scanning process of the participants and 3D foot and last models in the Foot3D software.

3D feet models are cut to similar ankle height and placed in the same position concerning the general coordinate system [24], as shown in Fig. 2. The longitudinal vertical section defining the XY plane is drawn through two main landmarks: on the pternion point (Pt) and the second toe [24,25]. More accurate results of location 2 use a landmark on the head of the second metatarsal bone (point 2).

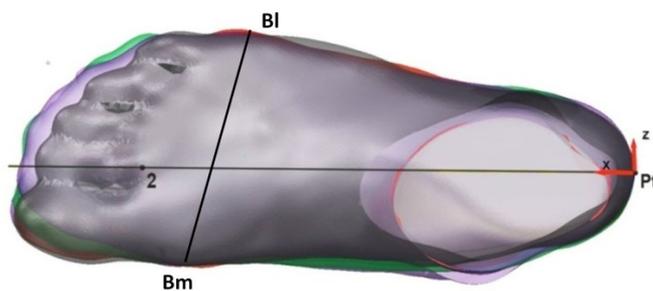


Fig. 2 Mutual alignment of the feet models for parameters comparison.

To compare 3D foot and last models, it is necessary to take into account the main anthropomorphic parameters of the foot involved in the shaping of the last. Foot and last models are set up to the appropriate heel height. The longitudinal axial section of the last (XY plane), which passes through the most convex point of the heel contour and the center of symmetry of the toe part, coincides with the foot axis, which passes through the pternion point and the head (middle of the base) of the second metatarsal bone, as presented in Figure 3. Table 1 depicts the results taken from the feet measurements.

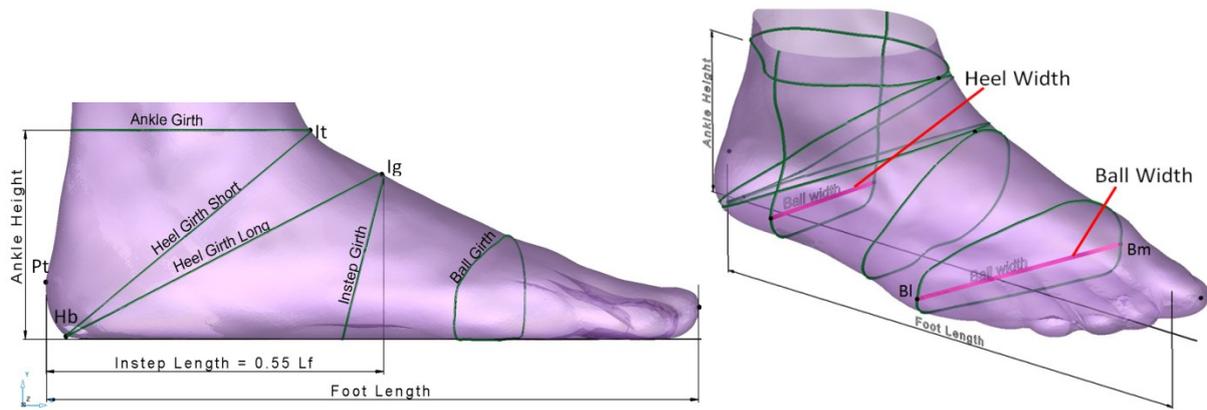


Fig. 3 Scheme of the foot measuring.

Table 1. Results taken from feet measurements.

Anthropometric parameter	Average value (μ)	Minimum (min)	Maximum (max)	Absolute diff. between min and max (Δ)	Standard deviation (σ)
Foot length (mm)	245.3	242.9	247.2	4.3	1.4
Ball girth (mm)	232.2	225.3	237.5	12.2	3.7
Instep girth (mm)	228.2	223.3	233.4	10.1	4.2
Ball width (mm)	93.2	91.6	96.3	4.7	2.0
Heel width (mm)	62.1	59.6	64.2	4.6	2.0
Heel girth (long) (mm)	325	302.3	338.6	36.3	13
Heel girth (short) (mm)	307	296.5	322.7	26.2	11
Ankle height (mm)	76.0	72.5	77.8	5.3	4.1

The alignment of the 3D foot and last models along the bottom contour of the longitudinal-axial section is orientated to the points Pt and B0. Point B0 is chosen as the base point of lasts alignment and is the projection of point B to the horizontal plane. The choice of this point is based on the peculiarities of forming the last shape and functioning of the foot [26]. The front part of the last with different styles located further than the medial ball (B) from the heel point is equally oriented in space. Figure 4 depicts views from this process.

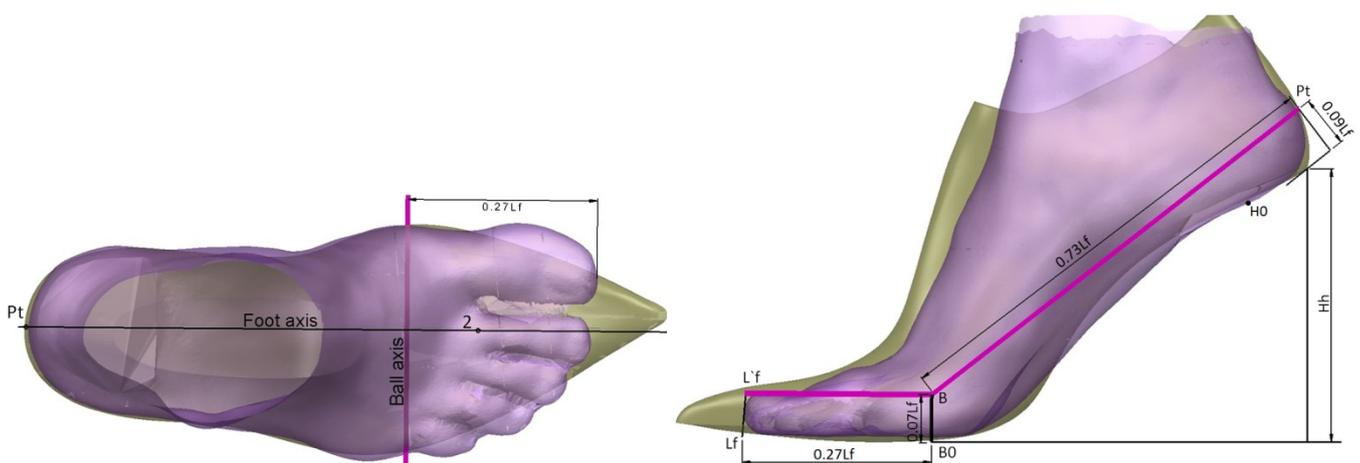


Fig. 4 Mutual alignment of the feet models for parameters comparison: a) top view and b) side view of the 3D foot and last models.

Changing the last height modifies only the position of the back part due to rotation by an angle that depends on the heel height. This is an important prerequisite for further designing of individual parts of

the last which can be interchangeable. During the process of combining different last models, the points B0 and two auxiliary points on the toe part of the bottom contour of the longitudinal section at a distance of 30 and 60 mm from B0 are used. To find point B0 at a distance of $0.73 L_f$ from the pternion point for lasts with the heel lift, it is necessary to use a neutral base of the foot passing through point Pt, point B, and point Lf. Figure 5 depicts the alignment of the last models with respect to the common coordinate system.

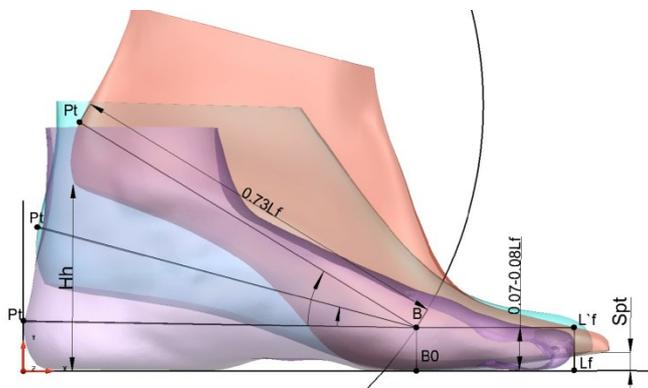


Fig. 5 Alignment of the last models with respect to the common coordinate system.

3 Results and discussions

3.1 Calculation

A set of typical cross-vertical sections on the major anthropometric landmarks is used to evaluate and compare the geometric parameters of the complex three-dimensional shapes of last and foot models. For this purpose, aligned models of feet and lasts are intersected by transverse vertical planes equally arranged at the level of the major anthropometric points. The widths and heights of each section are determined as depicted in Figure 6. Meanwhile, Table 2 presents the correlation of the basic parameters of customers' feet and their corresponding lasts based on the example of five orders. Shapes of these lasts are presented in Figure 7.

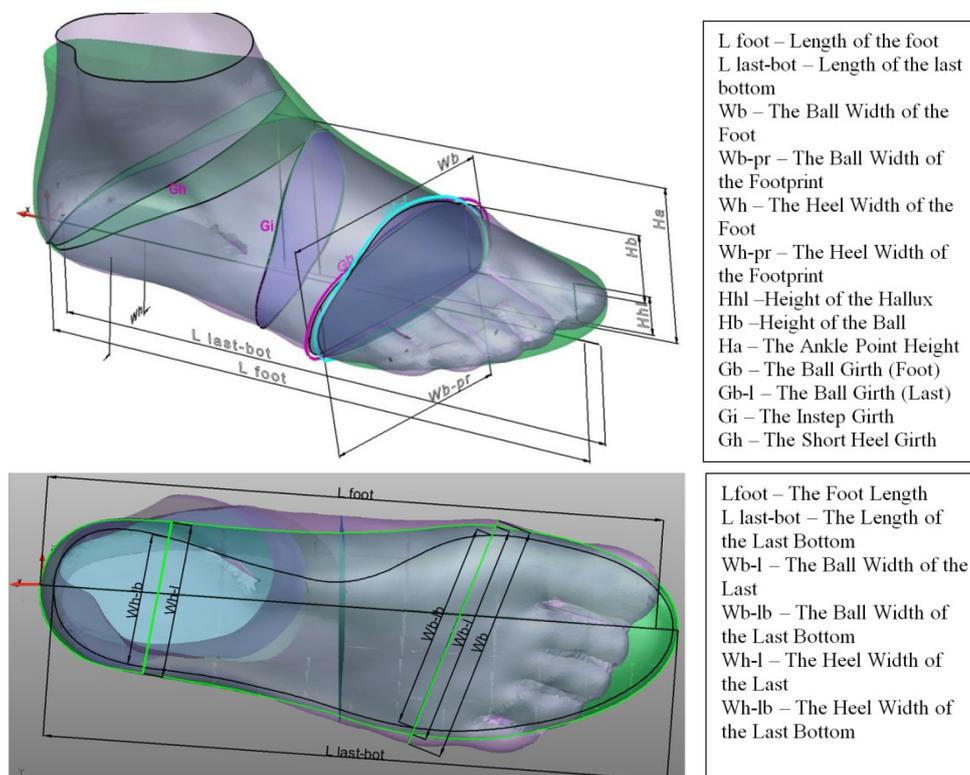


Fig. 6 Determination of basic foot measurements and shoe last parameters for comparison.

Table 2. Basic parameters of customers' feet and their corresponding lasts.

Feet Parameters	L – Length (mm)	Wb – ball width (mm)	Wh – heel width (mm)	Gb – ball girth	Gi – instep girth	Gh – short heel girth	Hhl – hallux height	Hb – height of the ball	Ha – ankle height
Foot 1	246.5	91.6	62.3	230.9	232.4	307.7	21.1	27.8	73.4
Last 1	255.1	83.5	61.5	224.1	242.4	322.0	21.5	28.1	80.6
abs. dev.Δ	8.6	-8.1	-0.8	-6.8	8.0	14.3	0.4	0.3	7.2
Foot 2	246.1	92.8	60.5	235.0	228.3	299.5	20.5	29.0	70.1
Last 2	261.0	85.4	60.0	227.2	238.1	321.0	21.3	28.2	80.5
abs. dev.Δ	14.9	-7.4	-0.5	-7.8	9.8	21.5	0.8	-0.8	10.4
Foot 3	244.6	92.4	61.9	231.1	224.2	303.5	20.8	31.5	72.9
Last 3	251.5	85.2	60.8	225.5	239.5	325.5	22.6	30.8	84.8
abs. dev.Δ	6.9	-7.2	-1.1	-5.6	15.3	22.0	1.8	-0.7	7.9
Foot 4	244.2	95.9	64.2	237.5	232.9	312.7	19.5	28.2	72.5
Last 4	263.5	86.4	62.5	225.3	240.8	320.5	18.9	26.5	83.4
abs. dev.Δ	19.3	-9.5	-1.9	-12.2	8.1	7.8	-0.6	-1.7	10.9
Foot 5	245.5	93.5	59.6	226.3	226.6	311.8	19.2	26.8	74.7
Last 5	264.4	85.7	58.9	217.9	232.1	328.6	18.5	25.5	88.6
abs. dev.Δ	18.9	-7.8	-0.3	-8.4	5.5	16.8	-0.7	-1.3	13.9

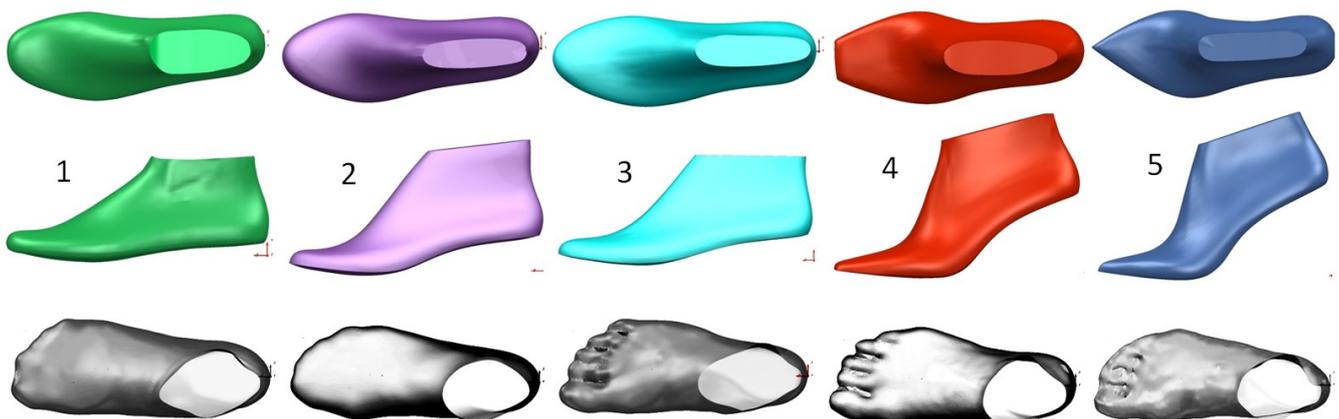


Fig. 7. Lasts shapes for five individual orders from Table 3.

The geometric parameters of the feet and lasts in the typical anthropometric areas show different and irregular absolute deviations Δ among the corresponding parameters. This is especially in terms of the girth parameters from different styles of lasts. As the differences are acceptable from the point of view of foot physiology, it is possible to find a section that can be averaged among all objects under study without affecting the comfort of certain samples. The cross-sectional contour separating the main part and toe part must be the same for various lasts. Moreover, to find the parameters of this section, it is necessary to analyze the shape and parameters of the studied feet in different areas of the metatarsophalangeal region and the difference in the parameters of the corresponding lasts in a similar area. In addition, when bringing the last of different shapes to uniform parameters in a certain cross-section, it is necessary to proceed from the fact that the location of this cross-section should correspond to the foot region with the lowest value of parameters deviation based on anthropometric measurements.

The bones and joints mobility is the greatest in the toe region, which contributes to the maximum deviation of the parameters of the lasts from those of the feet, which is allowed within the rational values [27]. This indicates the possibility of a union of the parameters of various lasts in this region for the purpose of standardization of individual parts, contours, and sections. Each shape of the toe part used

for the development of new lasts can be standardized within the closest parameters of lasts for customers without significant pathologies and deformities of the toe region.

3.2 Creating variants of last shapes

The preliminary general measurements show that the width parameter measured through the inner and outer balls ranges varies from 91.6 mm to 96.3 mm. The girth parameter of this area ranges from 225.3 mm to 237.5 mm (Table 1). In the area of the first phalanges of the toes, this difference is reduced, and the difference in the parameters of the foot profiles is similar.

To identify the area with the most similar parameters of the studied 3D last models, at first all last models are aligned and then compared in pairs using the Comparative Analysis feature, as presented in Figure 8.

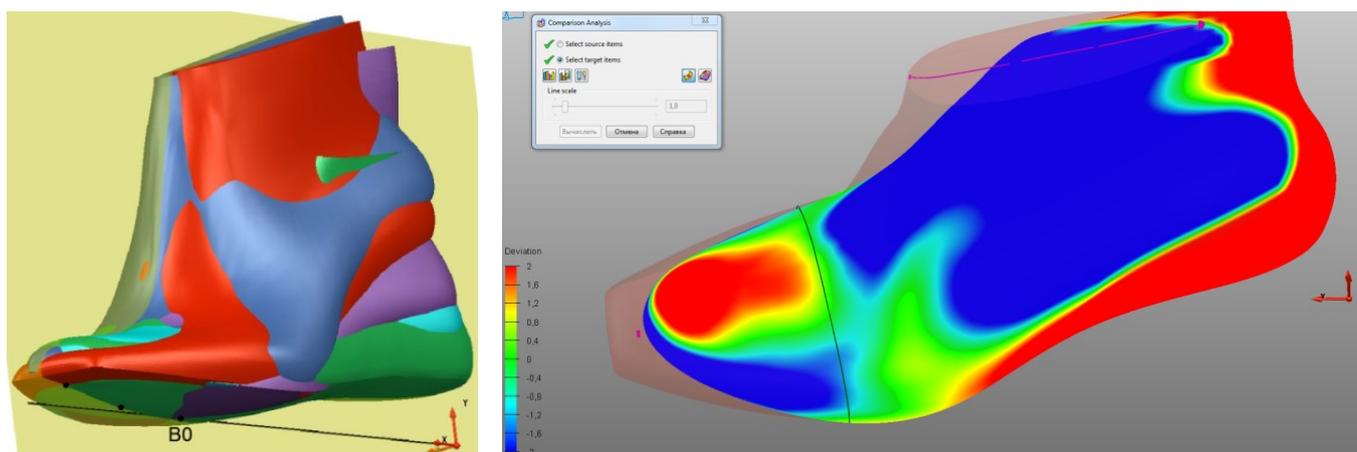


Fig. 8 Comparative analysis of the shoe last models in PowerShape CAD.

The results indicate that the areas with the smallest deviation are between the sections of 0.73 N and 0.8 N from the pternion point. Evaluating the difference between parameters and models configuration of feet and lasts, six transverse vertical sections through the inner and outer balls are taken at the distance of 0.73 N, 0.76 N, and 0.8 N from the pternion point perpendicular to the longitudinal axis of the last and at a certain angle in relation to it as depicted in Figure 9.

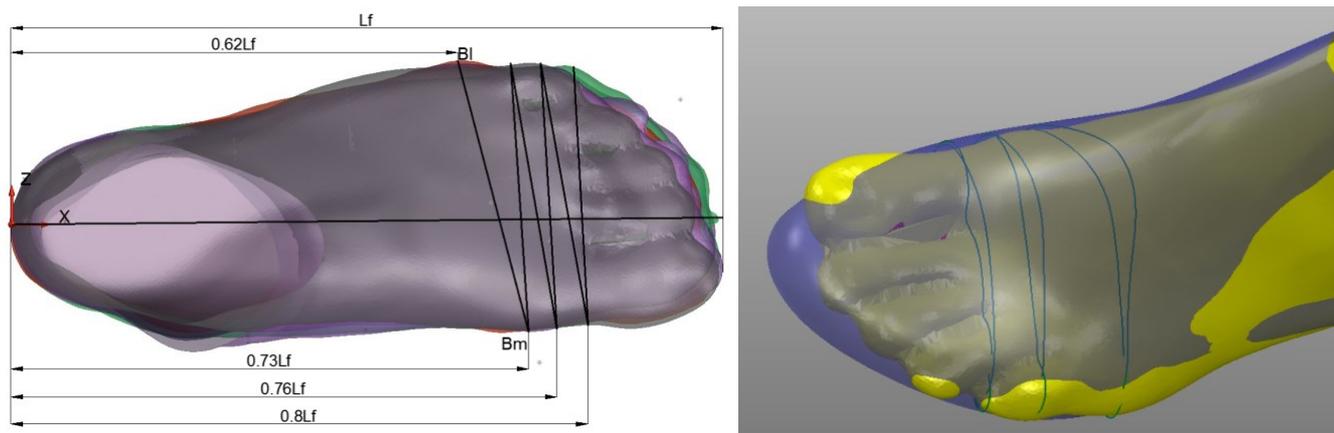


Fig. 9 Construction of cross-sections in the area of the search for a rational arrangement of the plane of last cutting.

Table 3 depicts the results taken from measuring the cross-sections, mean values, and standard deviations for the width and girth of each section of the foot, and last models.

Table 3. The results of measuring the cross-sections of 3D foot and last models. All values are given in mm.

Sections parameters (W-width, G-girth)	Foot					Last				
	μ	min	max	Δ	σ	μ	min	max	Δ	σ
W 0.62/0.73	93.2	91.6	96.3	4.7	2.0	86.0	83.5	88.0	4.5	1.4
W 0.73	90.8	89.6	91.9	2.3	1.0	85.2	83.1	86.8	3.7	1.1
W 0.73/0.76	90.2	88.9	91.1	2.2	0.6	84.1	82.9	85.5	2.6	0.8
W 0.76	88.4	86.9	89.7	2.8	1.1	82.7	80.9	84.8	3.9	1.3
W 0.76/0.8	88.0	86.8	90.4	3.6	1.3	82.0	79.7	84.2	4.5	1.6
G 0.62/0.73	232.3	225.3	237.5	12.2	3.7	222.9	217.9	226.4	8.5	2.9
G 0.73	222.2	219.6	225.8	6.2	1.7	212.4	210.2	215.6	5.4	1.7
G 0.73/0.76	216.8	214.2	218.5	4.3	1.6	211.9	209.8	214.0	4.2	1.3
G 0.76	211.8	206.5	214.6	8.1	3.0	206.3	205.1	209.0	3.9	1.9
G 0.76/0.8	202.9	200.1	208.8	8.7	3.7	198.4	195.8	202.3	6.5	2.1

Comparing contours of the respective sections of the lasts revealed a different configuration of the contours, even with very close parameters and small root-mean-square deviations of the basic parameters. This is because the basic lasts used in the reverse engineering process to design new personalized shapes have a different, non-uniform configuration and are designed arbitrarily, as art forms. As can be seen from Table 3, the standard deviation (σ) of the width and the girth parameter of the 3D model has a minimum value for both feet and last models at the level of the cross-section of 0.73\0.76Lf. Variation of the parameter is within the physiological interval of indifference, i.e., such a value does not affect the physiological comfort of the customer. Therefore, to unify this area of the lasts, we can average this section and its dimensions without compromising the ergonomic characteristics of the lasts. In this area, a sectional plane of segmentation is proposed to place that divides the last into two parts, the main – back – part and the replaceable toe – forepart.

Despite the variety of foot shapes and the futility of attempts to unify the base parts of a last, especially in the case of piecework production, it is assumed that is possible to achieve unification of a certain area of the last surface within the same size and width, which will have similar parameters for different foot shapes and styles of the toe part. But at the same time, for the cutting plane it is also necessary to take into account the peculiarities of changing the foot configuration when standing in high heels to ensure the possibility of multi-purpose use of the removable toe part.

Due to the changes of the foot in the area of the first phalanges of toes standing with or without high heels, the cutting plane is placed behind the area of the balls. When designing a shoe last for different heel heights, a variation in the parameters and shape of the back part is caused by foot biomechanics and a change in the position of its joints. The change in toe parts of lasts with different styles results from design peculiarities. However, in the border zone, the area with the parameters variable to the minimum extent can be found. The results taken from 3D foot models reveal a decrease in the girth of the foot in zones A and U by 4-5 mm with an 8 cm heel rise (Fig. 10). At the same time, the results of manual measurements show that the change in the girth of the foot in the U zone comprises 3-3.5 mm, which does not exceed the physiological indifference interval. Here, the position of the cross-section in the U region remains almost unchanged by increasing heel lift height.

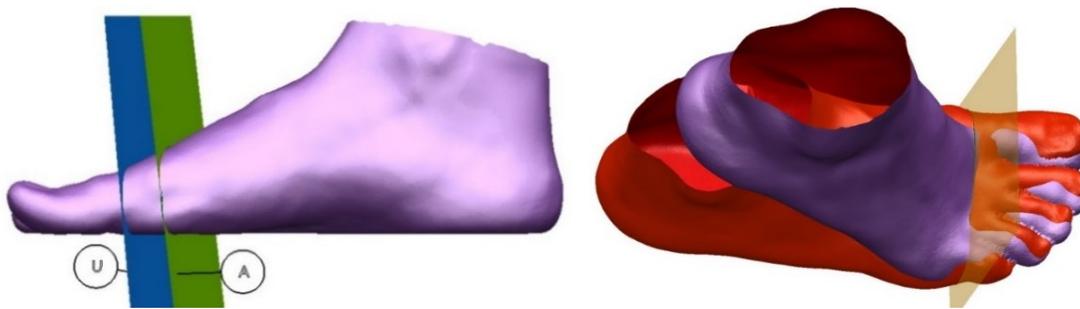


Fig. 10 Position of cross-section corresponding to the unchanged area of the foot.

Comparative analysis of two foot models confirmed the minimal deformation of zone U of the foot with an increase in the heel lift (Fig. 11). The foot width in this zone varies from 1 to 1.5 mm, which is almost within the measurement accuracy. So, section U is the most suitable for placing the zone of joining two parts of different last shapes. These results provide the basis for the development of a last with the convertible toe part, where the back and front parts of different lasts can be effectively interchanged to create a new shape. The sectional plane for the shoe last is drawn through points F1 and F2, as depicted in Figure 12.

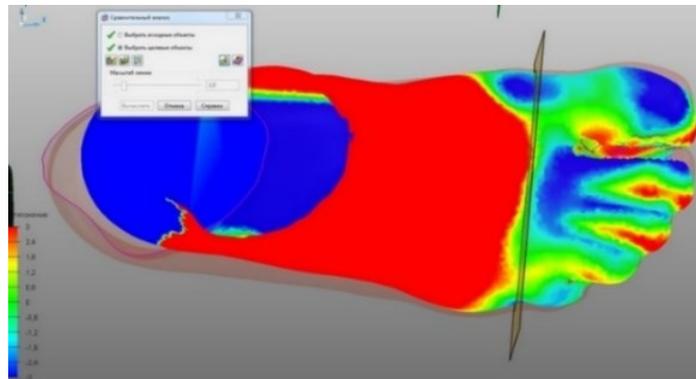


Fig. 11 Comparative analysis of the foot models with heel lifts 0 and 80 mm.

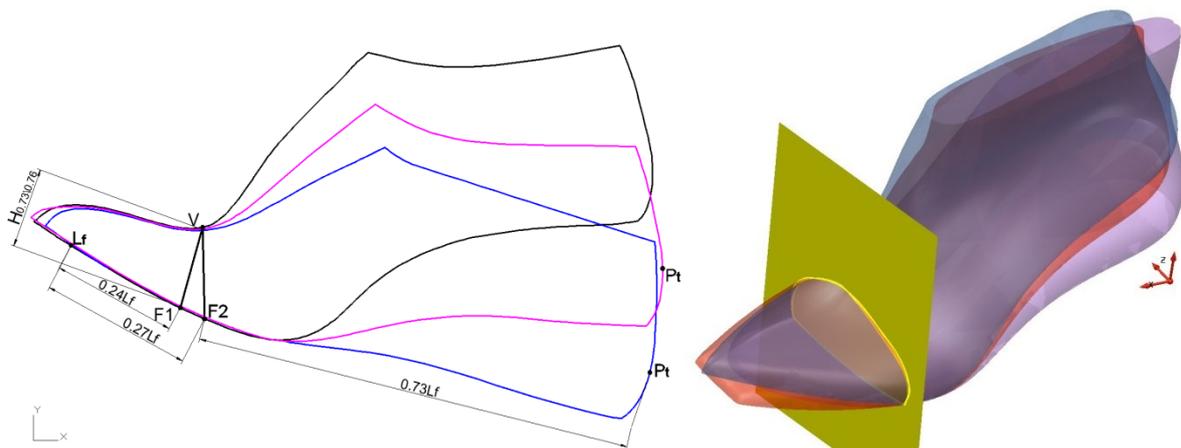


Fig.12 2D and 3D view of aligned lasts with a common sectional plane.

Developing such basic last shapes that can have a common unified cross-section in the specified area is achieved by modifying the studied last models using 3D modeling features of PowerShape and Crispin LastMaker software packages [28,29]. Here, lasts are compared with feet models and other lasts (Fig. 13). At the same time, we pursue the goal of modifying lasts only in the area adjacent to the joining area so that the main part of a last developed for a specific customer remains unchanged.

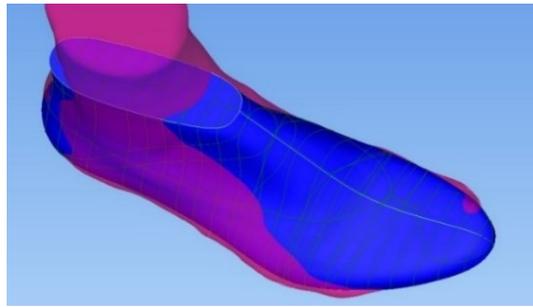


Fig. 13. Designing the shoe last shape in the mode of comparison with the foot

To unify the shape of the joining zone of the two parts of lasts, templates of the bottom surface are used. They have the same measurements in the 0.76/0.73 Lf section (Fig. 14). The shape of the last was adapted to the template of the shoe's last bottom [30]. Using the mode of comparison with the foot, a rational configuration of section 0.76/0.73 Lf was determined based on which the template was developed. After cutting the last along the specified section, the resulting template was used for all models of lasts. Figure 14 depicts the bottom contours of the lasts that were designed by uniform parameters of the cross-section 0.76/0.73 Lf.

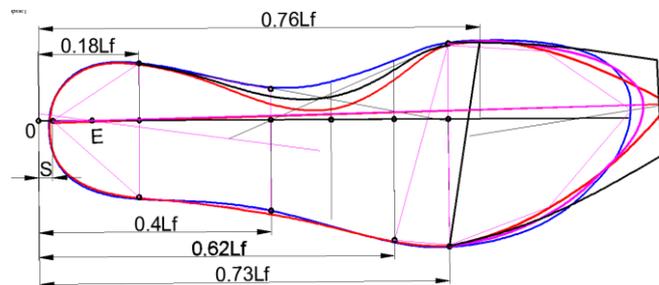


Fig. 14. The last bottom contours.

For a complex irregular shape of a shoe last, there is a problem of joining separate areas of different shapes. Several publications are devoted to this problem, in which authors suggest the use of blending and morphing functions. In Ref. 31, a study of the geometric continuity of surface areas by a shape-blending process was developed. This paper studies the continuity of the ruled surfaces constructed by linear interpolation between two pairs of continuous curves. Another study introduced a method that allowed two objects to be morphed using variable interpolation [32]. Following the method to join two areas of the shoe last shape based on the interpolation of morphing and blending to automatically obtain a smooth filling surface can be conducted in a controlled way [33]. Another technique proposed is based on the use of curve networks that are created on the surfaces to be joined, instead of using discrete data [34].

In this work, due to the convenient combination of shapes with a similar cross-section in the joining area, the morphing method is chosen. Figure 15 depicts the use of morphing for adjusting the area of the surface of the main part of the last to the contour of the joint.

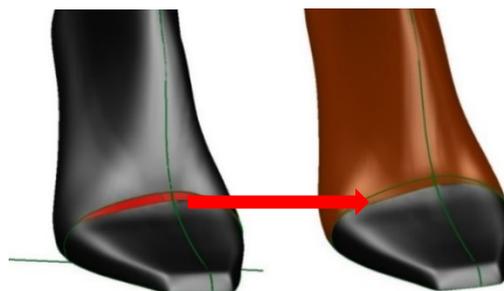


Fig. 15. During the morphing process of the contour of the joint.

By applying a unified contour of the sectional plane for several lasts with similar physical parameters of the main part, it is possible to increase the existing range of lasts' shapes by x times (x is the number of initial lasts with different shapes of the toe part) by clipping and replacing the toe part, as depicted in Figure 16. The initial last shapes are marked with the same colors.

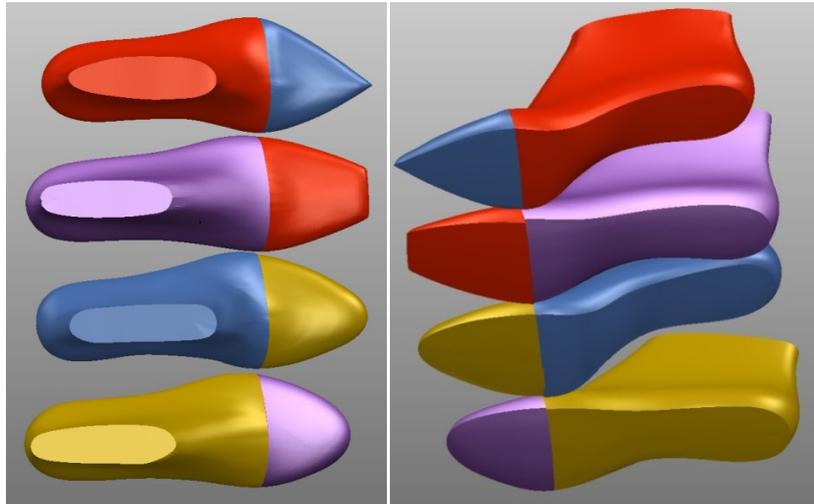


Fig.16 Variant of the extended range of last shapes resulting from the replacement of the toe part.

The investigation continued to evaluate the possibility of interchanging individual parts of different lasts using the example of lasts made based on the designed 3D models with CNC machines. Manufacturing a new pair of lasts every time for the sake of changing the shape of the toe part is impractical. But applying the method proposed in this work reduces material and costs. This is especially important today when the problem of overproduction of goods is so acute in the world. For the practical implementation of the proposed method, two shoe lasts of the same size but with different heel heights and toe parts are chosen:

- 1) with heel height 2.5 cm and round toe part
- 2) with heel height 0.5 cm and oval toe part

A CNC milling machine is used for the last production with two replaceable toe parts, as depicted in Figure 17. 4 different shoe last shapes result from this process:

- Model 1 - with heel height 2.5 cm and round toe part
- Model 2 - with heel height 0.5 cm and oval toe part
- Model 3 - with heel height 0.5 cm and round toe part
- Model 4 - with heel height 2.5 cm and oval toe part

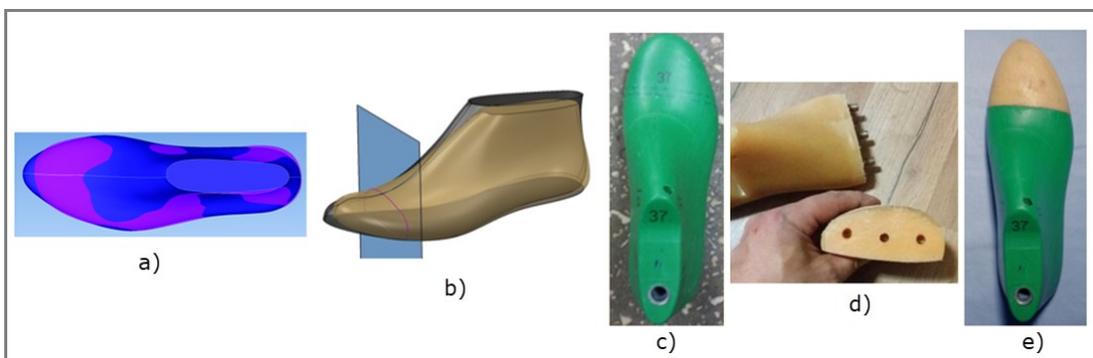


Fig. 17. The produced shoe last with a replaceable toe part: a) Comparison of the two 3D last models; b) section plane taken at last models; c) the last model of the first design; d) the second shoe last with the convertible toe part; e) the combined last shape.

After developing new last shapes, they are scanned and compared with the scanned foot models (Fig. 18), in order to check them for correspondence with consumers' foot shapes. Comparison of the foot model and the last showed a good correspondence level. Figure 18 depicts views of the data comparison between the resulting shoe last and customer's foot model.

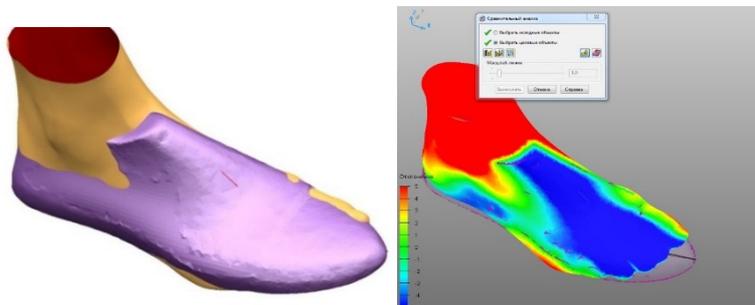


Fig. 18. Comparison of the resulted shoe lasts with the customer's foot.

Finally, the resulting shoe lasts with a replaceable toe part are used for shoe production. By following the relevant ecological fashion trend, textile materials are used for the upper part of the shoe produced by using the custom-made lasts with convertible toe part depicted in Figure 19. The customers showed positive feedback after their use.



Fig. 19. Shoe models produced with custom-made last: A – shoes made on the shoe last model 3, B - shoes made on the shoe last model 2, C - shoes made on the shoe last model 1.

4 Conclusions

In this work, a comparative analysis of selected 3D feet and last models of the same size, namely their forepart, to identify the area for possible averaging and bringing to common contours of the last was conducted. The proposed method of creating the new shoe last shape using two separate parts from different shoe last shapes allows expanding the range of last styles without increasing their physical number. This helps us to extend the shoe's last use, making it multifunctional from the design point of view. The segmentation plane divides the body of the last in such a way that the removable toe part can be attached to the main back part of the lasts of different shapes and styles. The use of graphic CAD software coupled with 3D scanners to obtain initial information makes it possible to effectively apply anthropometric information about the foot as a reference point when designing ergonomic lasts according to individual parameters. The use of numerous tools of software packages allows working with large volumes of 3D information, creating databases of feet and lasts, as well as using almost any method of transformation and modification of complex 3D last shapes. This solution provides limitless possibilities for creating new last designs with significant cost savings.

Author Contributions

Author contributions statement: L. Chertenko: conceptualization, methodology, software, validation, writing – original draft preparation; T. Spahiu: investigation, writing – review & editing; T. Lypskyi:

software, visualization; H. Almeida: writing – review & editing; O. Bondar: writing – review & editing. All authors have read and agreed to the published version of the manuscript.

Conflicts of Interest

The authors declare no conflict of interest.

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